Evaluating Ultimate Net Section Capacity

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Abstract:

This tech brief provides a succinct review of the structural parameters required to assess the risk associated with possible net section failure. The ultimate capacity of a load path is often evaluated employing the assumption of a perfect elastic/plastic alloy and the load redistribution based on the plastic section modulus.

Three parameters are reviewed. First, the difference between the elastic and plastic section modulus of a net section. Secondly, the implications that actual alloy properties have on the true stress/strain behavior compared to an idealized perfect elastic/plastic material. Thirdly, how displacement versus force controlled loading influences the ultimate capacity of a limiting net section. The review provides several reasons for considering the use of a plastic analysis when managing risk in features having low margins against full plastic moments as predicted with the plastic section modulus and a perfect elastic/plastic material model.

Plastic Net Section Finite Element Model:

Geometry:

A constant moment condition through a 1.00-inch diameter tube with a 0.061-inch wall is used to illustrate the potential benefits of employing a plasticity analysis when net section capacity is predicted to be potentially limiting. Figure 1 is a plot of the tube.

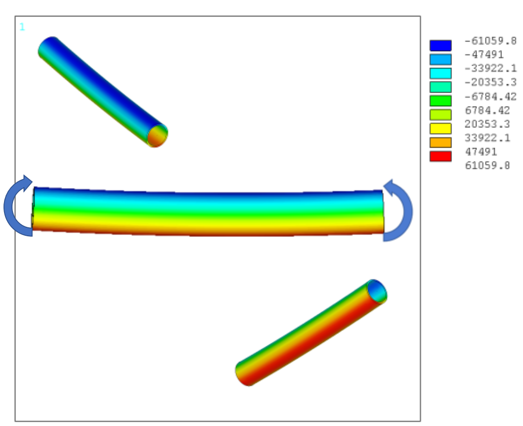

Figure 1 – Constant Moment Beam

Material Model:

The true stress-strain behavior at the beam's mid-span is evaluated for both force and displacement controlled conditions. The stress value of 61 ksi is just beyond the 0.2 percent offset yield of the alloy used in the analysis. Figure 2 provides the Ramberg-Osgood model of the alloy along with the idealized perfect elastic-plastic model. A kinematic hardening flow model is used in the plasticity analysis.

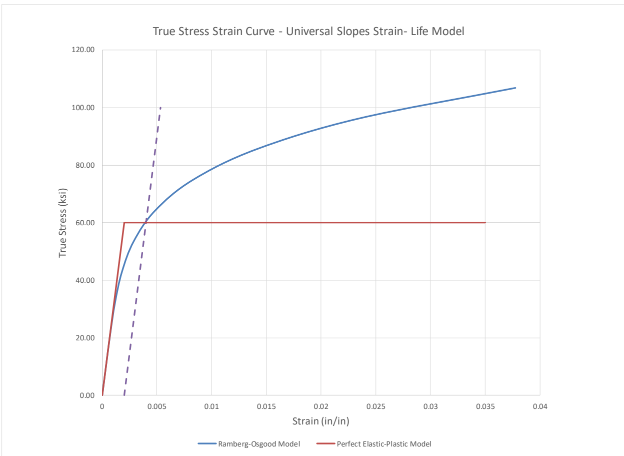

Figure 2

Net Section Moduli:

Elastic Section Modulus:

Figure 3 provides the second area moment of inertia and elastic section modulus for a hollow circular net section.

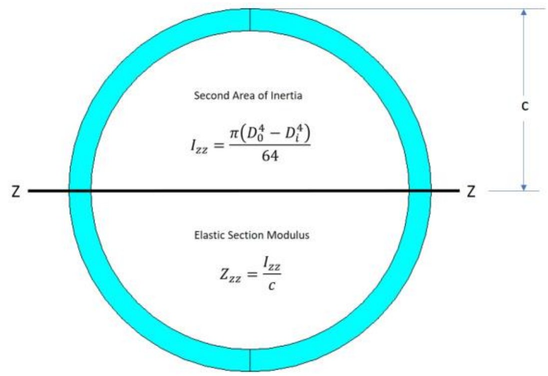

Figure 3

The second area moment of inertia is a means of evaluating the efficiency of cross sectional features within a net section which carry the resulting equilibrating flexural moment. The efficiency of a given unit area varies with the square of its distance from the axis about which the bending occurs. The resulting units for this parameter is length to the 4^{th} power (i.e., area x distance squared). The flexural stress varies linearly from a maximum in the area furthest from the neutral axis to a value of zero at the axis about which bending occurs.

Figure 4 provides a plot of this stress distribution. The elastic section modulus (i.e., Z_{zz}) is the second area moment of inertia divided by the distance from the neutral axis to the extreme fiber of the net section. The units for the section modulus is length to the 3^{rd} power. The maximum outer fiber elastic stress is the net section moment divided by the elastic section modulus.

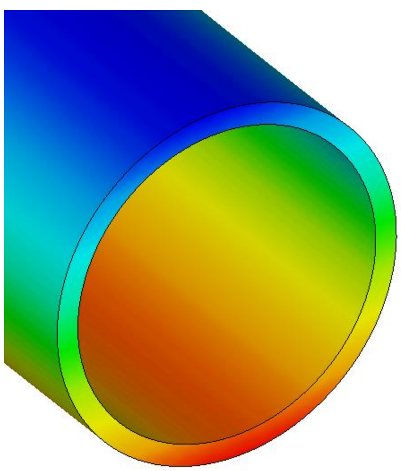

Figure 4

In the example, due to the direction of the applied moment, the stress on the top outer fiber is compressive (blue) and tensile (red) at the bottom.

Plastic Section Modulus:

It is apparent from the elastic stress distribution through the net section that not all of the area is equally contributing in equilibrating the moment carried through the section. When the section undergoes plasticity, however, the load that would have been carried through the most outer fibers is now redistributed to be carried by material closer in proximity to the neutral axis. The plastic section modulus is a means of computing the resulting capacity of the load path in flexure.

The plastic section modulus is the absolute sum of the first area moment above and below the neutral axis. When the section becomes a full plastic moment the couple is equilibrated through forces acting through the centroid of these areas.

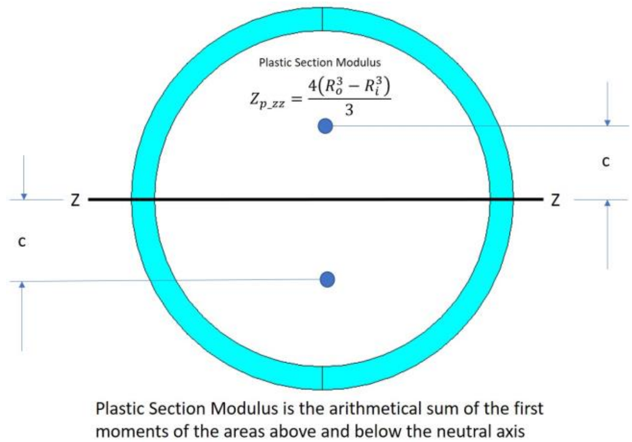

Figure 5 – Plastic Section Modulus

For the net section employed in the model, the plastic section modulus is 1.33 times greater than the elastic section modulus. Once the stress at the outer fiber exceeds yield for a perfect elastic-plastic material, the load will redistribute through the net section until the area above the neutral axis is in uniform compression and the area below the axis is in uniform tension.

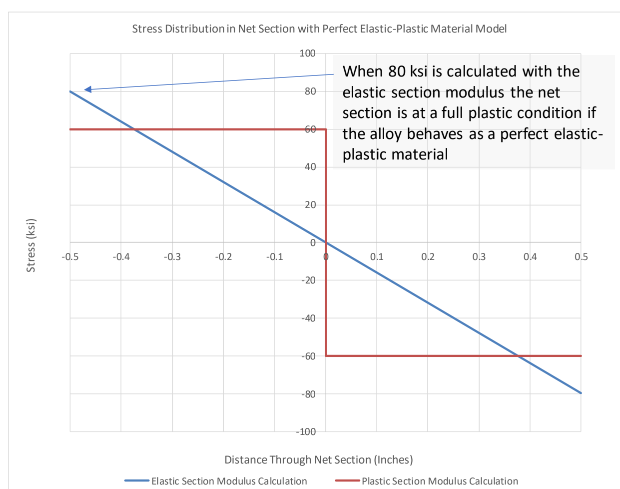

Figure 6

For a perfect elastic-plastic alloy, when the peak outer fiber elastic stress is calculated to reach 80 ksi the actual stress distribution would follow the red line in Figure 6. This creates a full plastic moment and theoretically any greater moment would produce an unstable condition. The alloy, however, does not behave in a perfectly elastic-plastic manner and therefore does not necessarily reach a full plastic moment under this loading condition.

Plastic Analyses:

A means of obtaining a more realistic evaluation of the ultimate net section load path capacity is with a plasticity analysis. If the alloy is cyclically hardening, then the true stress/stain monotonic behavior of the material should be used in the evaluation and if cyclically softening then the stress/strain behavior associated with the softened condition should be employed.

Several aspects of net section capacity, which are not readily available with a simple plastic section modulus calculation, can be captured in a plastic analysis. For example, the influence of displacement rather than force controlled loading is accounted for as well as potential local buckling which may control the load path capacity.

Displacement versus Force Controlled Loading:

Displacement controlled stresses are often referred to as secondary stresses. The reason is that the load tends to drop off as the displacements are maintained constant and plasticity is initiated. This results in lower maximum net section stresses than those generated with an equivalent force controlled scenario.

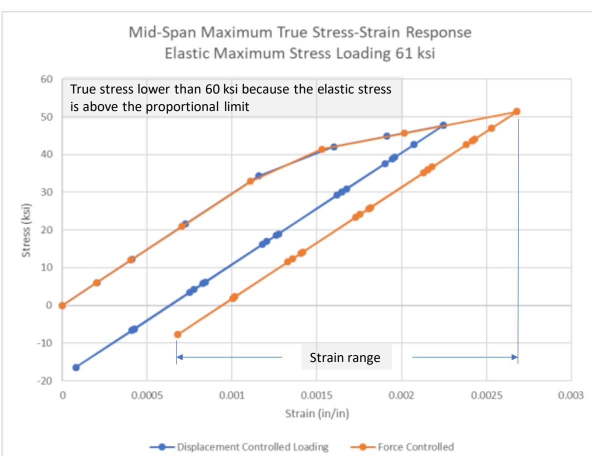

Figure 7

Several things can be observed from the true stress-strain behavior shown in Figure 7. First is that the true stress does not reach the yield value of 60 ksi. The reason is that plasticity begins to occur once the proportional limit has been exceeded. Depending on the alloy, the proportional limit can vary substantially from the 0.2 percent offset yield value. For the alloy used in the example, the proportional limit is approximately 0.67 of 0.2% yield. Reference Figure 2 for the true stress strain monotonic curve.

Secondly, it should be noted that the strain ranges between the two loading scenarios are virtually the same. Although the peak net section stress is 8% lower for displacement controlled loading, the cyclic strain range, which controls fatigue behavior, is similar in magnitude to the force controlled scenario. The only potential benefit in fatigue, for the displacement controlled condition, is that the mean stress is somewhat lower. This would be a secondary effect.

Figure 8 is the true stress strain response for both displacement and force controlled loading which, for an elastic analysis, would create a maximum elastic stress of 92 ksi at the outside fiber. Using a perfect elastic-plastic analysis this condition would be predicted to be unstable.

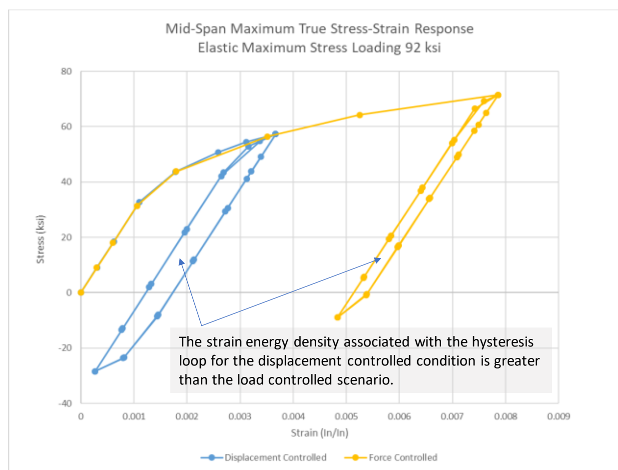

Figure 8

As with the lower level loading condition, the displacement controlled scenario creates a lower maximum net section stress. The peak displacement controlled stress is approximately 10% lower than the force controlled condition.

At this level of loading, however, the strain energy density associated with the hysteresis loop is significantly greater for the displacement controlled scenario. It would be anticipated that the fatigue life would be lower for the displacement controlled loading scenario. The only parameter potentially offsetting the effect of the higher cyclic strain energy density is the mean stress effect.

The reason that the displacement controlled loading produces a higher cyclic strain energy density is the additional axial load created when the enforced cyclic displacements remain constant and the geometry changes as plasticity in the load path occurs. This is seen in the stress distribution through the mid-span net section in Figure 9.

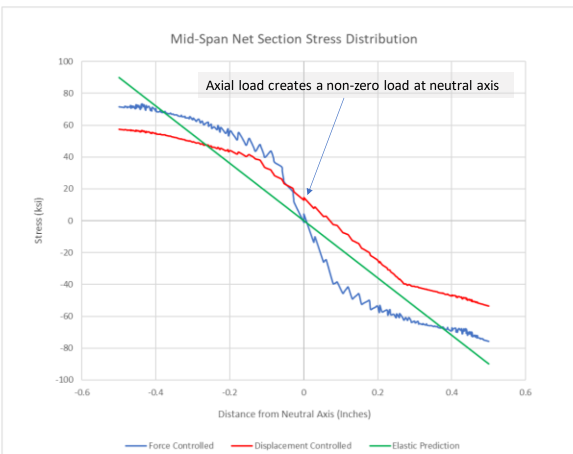

Figure 9

The plastic analysis captures the change in loading under displacement controlled conditions as significant net section yielding develops. This non-linear phenomenon cannot be accounted for with classical plastic section modulus calculations.

Local Stability Issues:

One other phenomenon that a plasticity analysis will address is the potential of local membrane buckling in the walls of hollow cross sections. Correctly employing plasticity in a finite element analysis requires accounting for large deflection and strains. A large deflection analysis updates the stiffness matrix based both on the geometry and material properties changes as plastic flow occurs.

Since the characteristic wall thickness in this example is less than 10 (0.5/0.061 = 8.2), the tube is not considered thin wall and net section rupture is more likely to occur before membrane crippling. If, however, membrane buckling was the limiting condition a plasticity analysis would capture the behavior as the solution attempted to converge.

Conclusions:

Employing plasticity analyses to evaluate net section capacity in limiting load path features provides an efficient means of accounting for both the residual strength capability of alloys above yield as well as the difference in load redistribution between force and displacement controlled loading.

In displacement controlled scenarios, a plasticity analysis captures additional load changes associated with large deflection/strain behavior. It also provides a means of assessing potential instabilities in the net section as plasticity is initiated.

To assess the actual structural risk of net section failure, consideration should be given to employing a plasticity analyses when net section margins, based on plastic section moduli, approach a negative value and structural changes are not a viable option. With currently available numerical and computational capabilities, the benefit-to-cost ratio of employing a plasticity analysis is significantly high in regards to making the best decisions in operational risk management.